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Abstract
Standard histories of economics usually treat the "marginal revolution" of the mid-19th century as both supplanting the "classical" economics of Smith and Ricardo and as advancing the idea of economics as a mathematical science. The marginalists—especially Jevons and Walras—viewed Cournot's (1838) book on mathematical economics as a seminal work on which they could build. Surprisingly, the scientist, philosopher, and logician Charles S. Peirce discovered Cournot before the marginalist economists and possessed a deeper appreciation of his mathematical approach. While Peirce's contributions to economics are limited, the influence of economics on his philosophy is subtle and not well understood. In a number of fragments, Peirce, who, despite Ricardo's lack of mathematical form, nonetheless regarded him as a paradigmatic mathematical economist, refers to "Ricardian inference" as a fundamental contribution to scientific method. Two options, perhaps complementary, are explored as to exactly what Peirce meant by "Ricardian inference." On the one hand, he associates Ricardo with the "primipostnumeral syllogism," which is a sort of generalization to uncountably infinite sets of what Peirce calls Fermatian inference (often referred to as mathematical induction). On the other hand, he holds up Ricardo as an exemplar of the "analytical method," which is Peirce's name for a hybrid form connecting analogy, abduction, and induction. On either account, economics plays a larger and more fundamental role in Peirce's philosophy of science than is generally understood. In the Harvard Lectures the two threads are linked together in Peirce's use of an economic example to exemplify pragmatism.

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Charles S. Peirce was an American original. Peirce scholars have noted his signal contributions to mathematics, logic, astronomy, geodesy, psychology, semiotics, probability and statistics, and other fields. It is less well known that Peirce was not only a student of economics but a widely read and unusually perceptive student. Carolyn Eisele, who deserves the most credit for bringing Peirce's few, previously unpublished economic writings to light, concluded that "[i]t is apparent that Charles Peirce played a role in the attempt to find new mathematical approaches to the investigation of economic theory" (Eisele 1979: 253; see also 368). Eisele's judgment is too vague, in that she never clarifies the nature of Peirce's positive contribution. It is also too generous: Peirce paid close attention to the economists, but there is no evidence that the economists reciprocated, and there is no reason to believe that 19th century economics would have developed differently had Peirce never existed. Nevertheless, Peirce was more than a student. He contributed two notable pieces of applied economic analysis: a paper for the United States Coast Survey on the economy of research in 1879 (W 4: 72–78), which was probably the most sophisticated application of mathematical economics to any concrete problem written up to that time, and a trenchant economic criticism of the editorial position of *The Nation* on the Spanish Treaty under negotiation in 1884 (W 5: 144–148).[1] In contrast to Eisele, we read the relationship between Peirce and economics as importantly running in the other direction: economics contributed to Peirce's understanding of other matters.[2] The importance of Peirce's analysis of the economy of research to the philosophy of science came to be widely recognized only in the mid- to late 20th century (see Wible 1998, 2018).[3] We suggest that Peirce credited economics with a more fundamental importance in his account of logic and the methodology of science, owing to its role in what he called *Ricardian inference*.

Exactly what Peirce meant by "Ricardian inference" is a puzzle. The vicissitudes of Peirce's life resulted in many unfinished projects and a complex mass of partly completed papers and incomplete and disjointed notes. And developing his idea of Ricardian inference is one of the projects that suffered. We are, however, able to offer a defensible conjecture of how Peirce would have developed the notion. We identify two distinct threads in Peirce's use of the term "Ricardian inference": one associated with his views on the mathematics of reasoning about infinities, and the other with his advocacy of analytic inference as the logic of science. While not using the term "Ricardian inference" explicitly, we show that Peirce connects these two threads organically in the

Harvard Lectures of 1903 as part of his account of the meaning of pragmatism, using an economic illustration.

1. Cournot and Ricardo

Economics today is highly mathematized—indeed, it is the most mathematical of the social sciences. It was not always so. Commercial arithmetic played a role in many early tracts in political economy. Various attempts were made to render economic ideas using geometry or elementary algebra. But economics came late to mathematically sophisticated analytical tools. The decisive shift occurred with the advocacy of mathematics by two of the leaders of the so-called "marginal revolution"—William Stanley Jevons in England and Léon Walras in France—who advocated the overthrow of the "classical economics" of Adam Smith and David Ricardo.[4] The key planks of marginalism were, first, casting the decisions of workers and consumers as individualistic exercises in utility maximization, and second, and perhaps more importantly, framing those decisions, as well as the production and sales decisions of firms, in terms that invited the application of the differential calculus. From the marginal revolution on, economics was indisputably a mathematical discipline—readily incorporating new mathematical techniques and, sometimes—as, for example, in the cases of game theory and operations research—proving instrumental in their creation.

As important as the marginal revolution was to the history of economics, few historical developments are truly unprecedented, and Walras and Jevons both acknowledged Antoine Augustin Cournot (1801–1877) as having stood at the forefront of mathematical economics and, to some extent, as having anticipated marginalism. Cournot was a French mathematician, astronomer, physicist, probability theorist, and historian of science. In 1838, he published *Recherches sur les Principes Mathematiques de la Théorie des Richesses*, in which he systematically applied the differential calculus to economic analysis. Cournot can be thought of as a proto-marginalist.[5] Where the marginalists derived a demand function from the maximization of utility, Cournot simply took the inverse relationship of demand to price as a primitive and focused instead on supply. Applying the tools of the calculus, Cournot mathematized Ricardo's system and went substantially further by investigating what is now called the theory of the firm, showing sequentially how a monopolist, a duopolist, and higher order oligopolists would set prices and output. He showed how in the limit increasing the number of producers of a particular product would generate what is now termed "perfect competition"—the market organization in which each producer can take prices as beyond his own control and simply decide how much to supply given the market price. While the names came later, Cournot was the first to draw those mainstays of economic analysis, the supply and demand curves (Cournot 1838[1927], Figures 1 and 6, fold-out insert of diagrams; see also Humphrey 2010).

Although the *Recherches* was the most advanced economics text of its day, it, like Hume's *Treatise of Human Nature*, fell stillborn from the press. Virtually no one took notice of it. The first review appeared only 19 years after it was published (Cherriman 1857; see also Dimand 1995), and the second seven years later (de Fontenay 1864). And, again like Hume, Cournot later felt compelled to publish two reworked versions of his neglected masterpiece—leaving out the mathematics in the hopes of attracting a wider audience. Jevons and Walras discovered it only after their own work was well advanced; and, though they admired Cournot, their work does not really build on him. Only after marginalism was well established did Cournot receive serious consideration, ultimately contributing key ideas that have been incorporated into modern economics.

One reason that Jevons and Walras did not make more substantial use of Cournot's work is simply that, even though they themselves advocated mathematical economics, they were not up to Cournot's standard. Historian of economics Mark Blaug notes that "Walras had only the instincts and none of the techniques of a mathematician" (1997: 279). Jevons himself notes in the preface to the second edition of his *Theory of Political Economy* that "[e]ven now I have by no means mastered all parts of [the *Recherches*], my mathematical power being insufficient to enable me to follow Cournot in all parts of his analysis" (Jevons 1965: xxx).

Although it went unnoticed at the time, Cournot did have one group of mathematically competent readers: the members of the Scientific Club of Cambridge, Massachusetts, a group that included both Charles Peirce and his father, the Harvard mathematician Benjamin Peirce, as well other members of Harvard's intellectual elite. In December 1871—three years before Walras first mentions him in print and a year before Jevons had even obtained a copy of the *Recherches* (Jevons 1879, p. xxx)—the Scientific Club met to study him together. Charles Peirce, who appeared to have missed the meeting, remaining at his job with the U.S. Coast Survey in Washington, D.C., nonetheless prepared diagrams for his father's use in the meetings and corresponded with his father and with the astronomer, mathematician, and sometimes economist Simon Newcomb about Cournot. In these letters, as well as in a later letter to an upstate New York lawyer named Abraham Conyer and notes preserved among his papers, Peirce displays a mastery of Cournot's economics, offering careful exposition, as well as critical analysis, of key parts of the book. Peirce's reading of Cournot is the beginning of a longer engagement with economics that is reflected, among other ways, in his frequent use of economic examples to illustrate philosophical points.

If Cournot represented the *avant-garde* of economics early in Peirce's career, why does Peirce cast David Ricardo as the eponym of the particularly vital form of inference? In his *Principles of Political Economy* (1821[1951]), Ricardo, a rich, retired stockbroker and member of the British Parliament, provided the canonical statement of the classical political economy that dominated British economics before the marginal revolution. To Jevons, the essence of progress was to supplant Ricardo by embracing Cournot, "for that able but wrong-headed man, David Ricardo, shunted the car of economic science on to a wrong line ..." (1965: li). Jevons opposed Ricardo's focus on the labor theory of value and the emphasis on supply over demand, as well as regarding him as a nonmathematical economist. Cournot, on the other hand, saw himself as building on, but not replacing, classical economics; and, while aware of Ricardo's lack of mathematical form, nonetheless regarded him as a kindred mathematical spirit. "There are authors," Cournot writes,

> like Smith and Say, who when writing on Political Economy, have preserved all the beauties of a purely literary style; but there are others, like Ricardo, who, when treating the most abstract questions, or when seeking great accuracy, have not been able to avoid algebra, and have only disguised it under arithmetical calculations of tiresome length. (Cournot 1838[1927]: 3)

Peirce, as we will document presently, fundamentally agrees with Cournot, but is willing to go further. Ricardo, in his view, is a fundamentally mathematical economist. The details of Peirce's economics trace mainly to Cournot, but the spirit is Ricardo's. Peirce appears to view Ricardo's most important contribution to be Ricardian inference, but at the same time he leaves considerable ambiguity about the nature of that contribution. What exactly *is* Ricardian inference? Our goal is to answer that question. While the answer—given the complexity and incompleteness of Peirce's professional papers—cannot be univocal, we believe that it nonetheless casts significant light on Peirce's philosophy of science and on the relationship of his understanding of economics to it.

2. Ricardo's Theory of Rent

In his "Sketch of a New Philosophy" under the heading "Development of the Method," Peirce refers to "[t]he peculiar reasoning of political economy; the Ricardian inference" (W 8: 21, 1890). In an entry for the *Century Dictionary* (1889: 3,081), Peirce glosses "Ricardian inference" as "the mode of inference employed by Ricardo to establish his theory of rent" (quoted at W 8: 365; see Ricardo 1821[1951], ch. 2). As is well known to historians of economics, Ricardo acknowledged that the theory of rent, developed in his *Principles of Political Economy and Taxation*, was originally due to Malthus (Ricardo 1821[1951], especially p. 5).[6] The attribution to Ricardo, in which Peirce is not alone, is in part a recognition of the central role that the theory of rent plays in Ricardo's systematic analysis of income distribution and growth for a whole economy (Blaug 1997, ch. 4; Eltis 1984, ch. 6). Peirce also sees Ricardo as essentially the founder of mathematical economics. Ricardo

> may be said to have rediscovered the reasoning of the differential calculus and applied it to the theory of wealth. All the so-called mathematical economists have done nothing which was not quite obvious after Ricardo's examples of analysis. (R 1126: 22–23, n.d.; quoted in Eisele 1979: 369)

Peirce explains:

> The reasoning of Ricardo about rent is this. When competition is unrestrained by combination, producers will carry production to the limit at which it ceases to be profitable. Thus, a man will put fertilizers on his land, until the point is reached where, were he to add the least bit more, his little increased production would no more than just pay the increased expense. Every piece of land will be treated in this way, and every grade of land will be used down to the limit of the land upon which the product can just barely pay. (CP 4.115, 1893)

Rent for Ricardo is the excess product of the more productive land relative to the land at the margin—that is, to the land that is just productive enough to pay the wages and normal profits necessary to bring it into production.[7] All land generates the same wages per bushel of corn and the same profits per pound sterling of capital laid out, but the most productive land pays the highest rent and the least productive (the marginal) land pays no rent.

Despite the agricultural illustration, Peirce has already generalized Ricardo's message to apply to all forms of production. He reads Ricardo through Cournot's glasses: production is carried on to the point at which the value of the marginal product of a factor of production equals its marginal cost (Wible and Hoover 2015: 517). But this is quite in keeping with Peirce's reading of the history of economics:

> Ricardo carried the analysis of political economy to its highest pitch, and Augustin Cournot treated the subject mathematically (as Ricardo did substantially, too) in a book whose mathematical blunders do not really affect its principal conclusions. (R 1123: 27, n.d.; quoted in Eisele 1979: 369; see also Wible and Hoover 2015)

Indeed, the passage in which Peirce explains Ricardo's theory of rent moves on seamlessly to an analysis of the incidence of import duties in which he refers to consumer behavior in a manner than assumes implicitly the existence of demand curves—fundamental elements of Cournot's analysis, but never mentioned by Ricardo—reaching the conclusions, first, that consumers will not pay the full amount of the duty, and second, that duties are most effectively levied on goods on which "our demand is so influential that a small decrease in the demand will cause a relatively large fall in the price" (i.e., on goods that are now referred to as *price-inelastic*) (CP 4.115, 1893; see the related passage in Cournot 1838[1927]: 46–47).

The theory of rent is simply an illustration for Peirce of something analytically and methodologically more fundamental. But exactly what? The question is hard to answer, for Peirce outlined many projects in which the methodology of economics would have formed an important part but that were never completed. We are left with headings and fragments and are forced to conjecture exactly where Peirce would have ended up.[8] There are two defensible options for further defining what Peirce meant by "Ricardian inference." On the first, Peirce took the lesson of Ricardo's theory of rent to be principally a mathematical one; on the second, he took it to be a lesson in scientific logic. Either option is consistent with seeing Ricardo's theory of rent as an important illustration of inference in action. Although the connection between these two options is not obvious at first, we will show that by the time of the Harvard Lectures of 1903, Peirce sees them as deeply connected.

3. The Primipostnumeral Syllogism

Peirce had a very broad conception of logic as the study of the principles that make our reasoning in all its many forms secure (CP 2.1, 1902). The never-completed "Qualitative Logic" (W 5: 323–371, 1886) was not devoted to reasoning in this broad sense but only to deductive inference, including mathematical deduction. In its table of contents, only two proper names appear: those of Fermat and Ricardo. "Ricardo's Inference" (chapter IX) sits between "Fermatian Inference" (VIII) and "Infinity & Continuity (X)" (W 5: 323). If Ricardo were one of Peirce's scientific heroes, Fermat would have been a demigod. Pierre de Fermat (1601–1665), the proof of whose "last theorem" eluded mathematicians for 325 years, was to Peirce undoubtedly an important mathematician, but, more than that,

> as a reasoner [he] cannot possibly be placed lower than second in the whole history of mind, for he invented a form of inference absolutely novel, and besides, discovered the mode of reasoning of the differential calculus, all but its notation.... (W 8: 269, 1892)[9]

Elsewhere, Peirce refers to Fermatian inference as the "greatest feat of pure intellect ever performed" (NEM IV: 87, 1892; see also Eisele 1979, p. 101; for a more detailed examination of Peirce's treatment of Fermatian inference, see Eisele 1979, ch. 14). And Ricardo stands shoulder-to-shoulder with Fermat in the plan for the "Qualitative Logic."

Fermatian inference takes the following form: Suppose that a series of numbers is indexed by $n = 1, 2, 3, \ldots$, that they are ordered by what Peirce calls a *generating relation*, and that they are conjectured to display a certain property (CP 4.198, 1897). If it can be shown that a) the property holds for some particular element n_0 in the sequence, and b) if it holds for some arbitrary element $k > n_0$, then it holds for the element $k+1$ as well, then the property holds for all elements in the sequence greater or equal to n_0. The inferential form, "as it is sometimes improperly termed, 'mathematical induction'," is a staple technique of number theory (W 8: 140, 1892).[10] Without Fermatian inference, which Peirce casts in a syllogistic form, "no progress would ever have been made in the mathematical doctrine of whole numbers …"; it "lurks beneath" the reasoning, even where it is not explicitly invoked (CP 4.208, 1897; see also NEM II: 43, c.1895).

Fermatian inference applies to infinite sets, but only to countably infinite sets, which Peirce and some modern mathematicians refer to as *denumerable* (CP 4.188, 1897). A set is *countable* when its elements can be placed in a one-to-one correspondence with the natural numbers (i.e., with the positive (or sometimes nonnegative) integers). In an early paper, the German mathematician Georg Cantor (1874) showed that some infinite sets cannot be placed into one-to-one correspondence with the natural numbers, so that some infinities are in fact larger (i.e., have a higher cardinality) than others. Peirce admired and accepted Cantor's analysis of infinities at a time when it was still deeply controversial among mathematicians.[11] Peirce referred to the smallest *abnumerable* (i.e., uncountable or nondenumerable) set as the *primipostnumeral multitude* (CP 4.200, 1897; NEM III/1: 82–96, c.1897). Mathematicians, including Peirce, indicate the cardinality of infinite sets by a sequence designated by the Hebrew letter aleph (CP 4.203, 1897). The cardinality of the countably infinite is $\aleph_0$, while that of the primipostnumeral set—that is, the first uncountable set—is $\aleph_1$.[12]

Fermatian inference does not work for uncountably infinite sets. The problem is that the primipostnumeral set cannot be constructed with the Fermatian generating function. Peirce illustrates the problem with the famous "paradox," in which in a footrace Achilles can never catch a slower moving tortoise, since every time he gets to where the tortoise was, it has moved on, and there remains a gap yet to close.

We can reconstruct Peirce's analysis (CP 4.202, 1897). Let Achilles start at position A_1 and the Tortoise some distance ahead at T_1. Achilles starts behind the Tortoise and runs faster. Peirce characterizes the position of the Tortoise according to a generating relation in which any point T_k is the point that the Tortoise has reached when Achilles reaches the Tortoise's previous location (i.e., when $A_k = T_{k-1}$. The Fermatian syllogism would then run as follows:

i) $A_k = T_{k-1}$, for all integers $k > 1$;	(generating relation)
ii) $A_1 < T_1$;	(particular property)
iii) if $A_k < T_k$, then $A_{k+1} < T_{k+1}$;	(conditional property)
$\therefore$ iv) for all integers $n > 1$, $A_n < T_n$.	Q.E.D.

The argument is correct: there is no point in the denumerably infinite set defined by the generating relation (what Peirce refers to as points in their *primal arrangement* (CP 4.198, 1897)) at which Achilles is ahead of the Tortoise.

But does the Fermatian syllogism show that Achilles can never catch the Tortoise? Not at all, says Peirce. The point at which Achilles catches the Tortoise—a point that is easily computed based on the differences between their two starting points and between their speeds—simply is not one of the points in the denumerable set defined by the generating relation in the Fermatian inference. Were we to add the catch-up point to the infinite sequence, we would still have a denumerably infinite set; yet "the inference does not hold" because the points "are no longer in their primal arrangement" (CP 4.202, 1897)—that is, the generating relation could not generate every point in the set. The essential problem is that the generating relation does not exhaust the points at which Achilles and the Tortoise can be located, which are nondenumerably infinite: "the denumerable collection in its primal order leads to no way of constructing or of conceiving of a primipostnumeral collection.... The primal arrangement of the denumerable collection affords no definite places nor approximations to the places for the primipostnumeral collection" (ibid.). In effect, Peirce argues that Fermatian inference restricts us to a subset of all possible numbers, and if that subset is chosen (i.e., if a generating relation is) such that it is defined to be those points at which the Tortoise is ahead of Achilles, then necessarily the point at which Achilles catches the Tortoise cannot lie in that subset (NEM II: 39–40, 147, c.1895). That says something about the nature of the subset; it says nothing about whether Achilles can actually catch the Tortoise.

There is no way consistent with Fermatian inference to state a generating relation adequate to describing the race between Achilles and the Tortoise, for the simple reason that space is continuous and an ordering based on the natural numbers necessarily cannot include the points designated by the irrational numbers that stand between any pair of rational numbers on the real line. Any adequate generating relation must be able to make sense of an orderly sequence of real numbers. Peirce argues that this is possible and offers a close analogue to Fermatian inference based on the construction of such an orderly sequence and of generating relations that employ it. He calls this inferential form the *primipostnumeral syllogism* and presents it in the form:

> Π_0, the unit of P_0, is X;
> If every Π of any pack is X, then every Π of the pack which is s of that pack is X;
> Hence, every Π is X. (CP 4.209, 1897)

To explain the details of the syllogism adequately would take us further into Peirce's mathematics than we need to go for present purposes. The interested reader can, however, find more details in the Appendix to the current paper. For now, it is enough to notice that the first line of the syllogism corresponds, with appropriate modifications to allow for continuous sets, to line ii), the particular property, and the second to line iii), the conditional property, of the Fermatian syllogism of Achilles and the Tortoise. The key innovation, however, is Peirce's claim to be able to state a sequential generating relation among the numbers in the primipostnumeral multitude (CP 4.207, 1897).

Peirce argues that the primipostnumeral syllogism is as central to the theory of real quantities as the Fermatian inference is to the theory of numbers, and even when it is not explicitly invoked, it "lurks beneath" reasoning using the differential calculus. In Peirce's hands, it is essential to the foundations of an approach to calculus grounded in infinitesimals as actual numbers with a real existence (NEM II: 147, c.1895). Infinitesimals already had a poor reputation in Peirce's day, having been pushed out of calculus by analysis in terms of limits. Peirce has "nothing against [the doctrine of limits], except its timidity or inability to see the logic of the simpler way" (CP 4.152, 1893). "[T]he hypothesis of infinite and infinitesimal quantities is consistent and can be reasoned about mathematically …" (CP 4.118, fn. 1, 1889; see also CP 3, ch. XVIII, 1900; and NEM III/1: 121–127, n.d.).[13]

Peirce does not claim to be original:

> I do not mean to say that the primipostnumeral syllogism is altogether unknown in mathematics; for the reasoning of Ricardo in his theory of rent, reasoning which is of fundamental importance in political economy, as well as much of the elementary reasoning of the differential calculus, is of that nature. But these are only exceptions which prove the rule…. (CP 4.210, 1897)

Unfortunately, Peirce does not give a detailed illustration of the application of the primipostnumeral syllogism to Ricardo's theory of rent. Again, it would divert us from our main purpose to dig deeper here. Nonetheless, in the Appendix, we conjecture a possible syllogistic argument for Ricardo's theory that appears to play by Peirce's rules.

Does Peirce in fact identify Ricardian inference with the primipostnumeral syllogism? His juxtaposition of Ricardian inference with Fermatian inference, the close cousin of the primipostnumeral syllogism, might point that way. And it is striking that of all the possible examples that might illustrate the syllogism, Peirce chose Ricardo's theory of rent. It is believable that Ricardo's vision of land taken into production sequentially in order of productivity could be captured in a generating relation suitably adapted to continuous quantities and the last land taken into production (i.e., the land that pays no rent) could be treated as an infinitesimal in Peirce's sense. Yet, in the passage in which he analyzes the primipostnumeral syllogism most carefully, Peirce never uses the term "Ricardian inference." Rather, he offers the theory of rent merely as an example. And in his exposition of the theory of rent, he neither mentions the primipostnumeral syllogism nor explicates the theory in a way that would highlight its operation. Instead, he provides an argument, which could easily have been taken from Cournot, drawing on ideas of the differential calculus that certainly predated Ricardo. To the degree that the primipostnumeral syllogism is foundational for the calculus, it is certainly involved in the theory of rent, but it is not clear that such involvement would warrant attaching Ricardo's name to the inferential form. Nonetheless, it illustrates the important point about Peirce's understanding of economics: it is essential to economics that it tap the deepest roots of science—its fundamental forms of inference:

> All the greatest steps in the progress of modern science have involved improvements in the art of reasoning. Harvey's discovery of the circulation of the blood, Kepler's researches on the orbits of the planets, Galileo's development of the principle of inertia, and many other such works contained lessons in logic.... This has been particularly the case with discoveries in mathematics. Pure mathematics, indeed, is nothing but an art of drawing conclusions of a particular description.... [T]he ideas of the infinitesimal calculus have penetrated everywhere, forming, for example, an important not to say the principal factor in Ricardo's political economy. (W 5: 325, 1886)

4. Analytical Inference

A second possibility for what Peirce meant by "Ricardian inference" is drawn from his broader conception of the logic of science. Nature is complex. Rather than attack it head-on, a strategy that has little hope of success, Peirce suggested that science ought to—and, in fact, does—address problems indirectly. The strategy is

> to substitute for those problems others much simpler, much more abstract, of which there is a good prospect of finding probable solutions. Then, the reasonably certain solutions of these last problems will throw a light more or less clear upon more concrete problems which are in certain respects more interesting.
>
> This method of procedure is that Analytic Method to which modern physics owes all its triumphs. It has been applied with great success in psychical sciences also. (Thus, the classical political economists, especially Ricardo, pursued this method.)[14] (CP 1.63–64, c.1896)

The context of Peirce's praise of Ricardo's methodology is a discussion of the history of science, and in particular its methodological lessons, in which Peirce frames the analytical method in the context of the logic of science (CP 1, ch. 1, especially sections 8–14 (paragraphs 61–85). Peirce famously and repeatedly throughout his career offers a tripartite classification of the fundamental types of logical inference (e.g., CP 1, ch. 2, section 10). He divides inference into, on the one hand, *explicative*, *analytical*, or *deductive* inference and, on the other hand, *ampliative* or *synthetic* inference (what is "loosely speaking" called *inductive* reasoning) (W 3: 297, 1878; see also CP 2, book III, part B). Deductive inference itself brings no new facts into our reasoning but works out the necessary consequences of facts already presumed to be true. Ampliative reasoning is divided into *induction* proper and *abduction*. ("Abduction" is Peirce's most common term for the latter form of inference, but at various times he preferred the terms *hypothesis* (e.g., W 2: 48, 1867; W 4:419, 1883), *retroduction* (e.g., CP 1.65 and 68, c.1896), or, more rarely, *presumption* (e.g., CP 2.774 and 791, 1901).

In an early exposition of the distinctions, Peirce relates the types of inference to the syllogism, starting with the classic form of *Barbara* (W 3: 325–326, 1878):

> DEDUCTION
> *Rule.* — All the beans from this bag are white.
> *Case.* — These beans are from this bag.
> ∴ *Result.* — These beans are white.
>
> INDUCTION
> *Case.* — These beans are from this bag.
> *Result.* — These beans are white.
> ∴ *Rule.* — All the beans from this bag are white.

Hypothesis [Abduction]
Rule. — All the beans from this bag are white.
Result. — These beans are white.
∴ *Case.* — These beans are from this bag.

Only the first inference is necessary: if its conclusion is false, one or more of its premises must be false. The second inference is not necessary—we may simply have failed to sample enough beans to find one of the black ones—but it offers us new knowledge, a *universal* conclusion about the contents of the bag. The third inference is also not necessary—the white beans could have come from some other bag—but it also offers us something new, an *explanation*. Much of Peirce's work as a logician was devoted to exploring the conditions under which the two unnecessary and therefore uncertain inferential forms would be substantively useful.

Peirce sometimes associates induction exclusively with statistical procedures with the determination of the ratio of predesignated quantities (CP 1.67, c.1896). For example, that the beans are either black or white predesignates the categories in which our results are to be classified, and the induction itself merely counts. The validity of the induction arises from a) *predesignation*—that is, the maintenance of the categories during the process of collecting and classifying the data; and b) *random sampling*—that is, following a procedure designed to make any group of elements of the population and any ordering of those elements as likely to be sampled as any other of the same size.[15] Without predesignation, there is a vicious circularity: we learn nothing new if we adjust the categories to fit the previously known data.

The essential advantage of random sampling is this: we may be mistaken with respect to the actual ratios or statistically determined values of a parameter, but if we persist we will either exhaust the population and therefore find the true value, or, in an open-ended population, converge on the true value according to known statistical laws (which are deductive truths) in a way that lets us set ever narrower bounds on the truth. It is a method, that, if persisted in, is inevitably self-correcting (W 4: 429, 1883; CP 2.781, 1901; EP 2: 216, 1903; CP 2.769, c.1905). Still, inductive inference is not necessary. For any finite sample, we cannot rule out that we got a bad draw and therefore that our best estimate is far from the truth. It is just that the probability of such a draw gets smaller as we proceed. What is more, inductive inference is conditional: the inevitable convergence to the true value occurs only if the framework of our predesignated categories is correct and the steps that we have taken to ensure random sampling are successful. The latter may be monitored statistically, but in both cases nothing rules out surprise: long after a stable ratio has been established between black and white beans, we may draw a yellow one, calling into question the predesignated categories and indicating that a new hypothesis is in order.

Peirce has no interest in ruling out surprise, so long as it is not so frequent as to strip induction of its usefulness, since surprise is the main source of new knowledge. Nature teaches us by a series of "practical jokes, mostly cruel" (EP 2: 154, 1903). The possibility of a surprise of the yellow-bean variety also shows that aside from the establishing of a ratio, induction sometimes provides the opportunity simply to test a hypothesis that has a yes or no, true or false answer. In fact, in some accounts of ampliative inference, Peirce includes under the heading "induction" the testing of hypotheses (the consequences of abductions), even when the hypotheses do not admit of a statistical characterization (W 2: 221, 1868; W 4: 439–441, 1883; EP 2: 205, 1903; CP 2.759, c.1905).

The theory of abduction is often seen as one of Peirce's most innovative contributions to logic.[16]

> *Retroduction* [*abduction*] is the provisional adoption of a hypothesis, because every possible consequence of it is capable of experimental verification, so that the persevering application of the same method may be expected to reveal its disagreement with facts, if it does so disagree. (CP 1.68, c.1896)

While induction can show us the need for something new, it is abduction that actually introduces it. Abductions take the form:

> The surprising fact, C, is observed;
> But if A were true, C would be a matter of course;
> Hence, there is reason to suspect that A is true. (EP 2:231, 1903)

Read in a bare way, abduction/induction (testing) seems to presage Karl Popper's conjectures and refutations or inference to the best explanation (Popper 1963, ch. 1; Lipton 2004; see also Töben 1986). Peirce calls abduction "inference," but in fact he does not object to calling it "conjecture" (EP 2: 231, 1903). Still, his view is not Popper's, for Popper famously denies that there is any scientifically interesting account of conjectures. Conjectures may spring from accident just as well as from as inspiration; from prejudice as from deep acquaintance; from mundane experience as from metaphysical poetry; from Marley's ghost as from "an undigested bit of beef, a blot of mustard, a crumb of cheese, a fragment of an underdone potato." Peirce accepts that abductive conjectures *could* come from anywhere, provided that they fit the appropriate inferential template. But simply guessing wildly in an infinite space of hypotheses would be a remarkably inefficient way to light on the truth. And he denies that ampliative inference fails, as Popper would later suggest, to have a useful logic.

To give one of Peirce's favorite examples, the game of twenty questions shows that a set of carefully structured hypotheses and tests can rapidly reduce the hypothesis space, so that only the truth remains: "twenty skillful hypotheses will ascertain what two hundred thousand stupid ones might fail to do" (EP 2: 109, 1901). But twenty questions or any other abductive strategy could not, Peirce believes, reduce the uncountable infinity of alternative hypotheses if people did not have at least a slightly better probability of picking the true hypothesis than mere random guessing (W 4: 445–446 and 447–450, 1883; CP 1.121, c.1896). The bias in favor of truth need not be high, but it must be there. Galileo and others who stood at the great junctures in the history of science relied on *il lume natural*, "an inward power, not sufficient to reach the truth by itself, but yet supplying an essential factor to the influences carrying their minds to the truth" (CP 1.80, c.1896; see also EP 2:32, 1898). Peirce's only explanation of the ultimate origins of our inferential good luck is that it is evolutionarily adaptive: survival in the kind of world we live in requires it (W 4: 447–450, 1883).

Abduction and induction are cooperative forms of inference, with abduction setting the framework for inductions and inductions evaluating abductions. Peirce goes further and suggests that there is actually a fourth, hybrid form of inference, which he terms "analogy" that combines the character of abduction and induction (CP 1.65, c.1896). It is analogy that forms the basis of the analytical method to which Peirce attributes so much scientific success in economics, as well as in the physical sciences. Peirce defines *analogy* as "the inference that a not very large collection of objects which agree in various respects may very likely agree in another respect" (CP 1.69, c.1896).

The definition itself illuminates very little. What Peirce appears to have in mind is something closely akin to the way in which modern economists, as well as scientists in many other disciplines, employ models as inferential tools. Self-consciousness about modeling in any science is largely a post-World War II phenomenon. Earlier, the word "model" in the sciences almost always referred to physical representations such as an orrery or a patent model. Yet there is good reason to think that it is not anachronistic to see the concept of modeling in Peirce's understanding of analogy as a distinct form of inference. Peirce does sometimes use the term "model":

> The word diagram is used here in the peculiar sense of a concrete, but possibly changing, mental image of such a thing as it represents. A drawing or *model* may be employed to aid the imagination; but the essential thing to be performed is the act of imagining. (NEM IV.219, fn. 1, n.d., emphasis added)

We will take up the role of diagrams as models in section 5 below.

Peirce takes *analogy* to be an English translation of Aristotle's *παράδειγμα*, which is the etymological source of *paradigm* (CP 1.65, c.1896). "Paradigm" is, in the wake of Kuhn's *Structure of Scientific Revolutions* (1962), widely used, but it was an obscure, largely grammatical term in Peirce's day, and Peirce would not have invested it with Kuhnian resonances. According to the *Oxford English Dictionary*, the root word *δειγμα* (*deigma*) designates a "sample [or] pattern," while the prefix *παρά-* (*para-*) conveys the idea of "analogous or parallel to, but separate from or going beyond" that pattern. The word itself aptly conveys the strategy of analogical reasoning that Peirce attributed both to physics and to classical political economy, and it maps very nicely onto modern practices in which stripped down or idealized root models are elaborated successively to come closer to empirical observations while maintaining their underlying basic character and tractability.[17]

Peirce clarifies analogy as a mixed type of inference with an extended analysis of Kepler's discovery of his laws of planetary motion: "the greatest piece of Retroductive reasoning ever performed" (CP 1.72–74, especially 74, c.1896). Kepler began with Copernicus's hypothesis of the planets in circular orbits around the sun and Tycho Brahe's and his own observations. The analogy was, if we can use the terminology anachronistically, between Kepler's mathematical model, with its precise orbits, and the actual observations. At first, the analogy was not a good one: the Copernican model fit the data rather badly. Out of keeping with Popper's later methodological pronouncements, Kepler did not simply scrap Copernicus's model. His procedure was not haphazard, but systematic and conservative, in the sense that at each new abductive step, he tried to preserve his quantitative success hitherto—that is, to stay within the bounds of error already achieved—and to use the specific ways in which the hypothesis fell short to suggest the next abductive step. Kepler's own abductive contribution was to consider the dynamical implications of the sun, which he knew to be vastly larger than any of the planets and which he conjectured exercised some vaguely defined causal power over them. Alternating abductions to introduce modifications and inductions to characterize the nature and degree of the deviations between conjecture and data, Kepler refined the model:

> [N]ever modifying his theory capriciously, but always with a sound and rational motive for just the modification he makes, it follows that when he finally reaches a modification—of most striking simplicity and rationality—which exactly satisfies the observations, it stands upon a totally different logical footing from what it would if it had

been struck out at random, or the reader knows not how, and had been found to satisfy the observation. (Ibid.)

The analytical method for Peirce is largely the method of refining and precisifying analogies.

Peirce divides the sciences of discovery into three: *mathematics* and, drawing on Bentham's terminology, *cœnoscopy* (in some places also called *philosophy*), and *idioscopy* (CP 1.239–242, 1902; EP 2: 259, 1903; Bentham 1816, pp. 177–179; Bentham 1952, p. 85). All these sciences are observational, though in different senses:

> Mathematics studies what is and what is not logically possible, without making itself responsible for its actual existence. Philosophy [cœnoscopy] is a *positive science*, in the sense of discovering what is really true; but it limits itself to so much of truth as can be inferred from common experience. Idioscopy embraces all the special sciences, which are principally occupied with the accumulation of new facts. (EP 2: 259, 1903)

All the special sciences are grounded in *cœnoscopy*:

> The analytical economics of Adam Smith and of Ricardo were examples of it. The whole doctrine in its totality is properly termed the Philosophy of Common Sense, of which analytical mechanics and analytical economics are branches. (CP 8.199, c.1905)

And all are open to idioscopic refinement.

The Copernican hypothesis of the planets moving in circular orbits about the sun was a piece of cœnoscopic astronomy, relying not on detailed observation but on what economists today refer to as "stylized facts." Kepler modified the analogy, but he also particularized and added precision to the facts, so that Kepler's laws are a triumph of idioscopic astronomy.[18] The root hypotheses of economics, such as the law of supply and demand, are similarly cœnoscopic: "In all economics the laws are ideal formulae from which there are large deviations, even statistically . . . general tendencies to which exceptions are frequent" (CP 7.158, 1902). Economic laws, for Peirce, are more true on average than they are for any individual. Economics is thus the natural landscape for statistical refinement, for inductions that raise the precision of the models (Peirce quoted in Eisele 1979: 251).

The essential point for Peirce is that analytical inference represents not—as Popper would have it—testing leading to relentlessly directing the arrow of *modus tollens* against a horde of wild hypotheses; rather, analytical inference represents a *constructive* interplay of abduction and induction. Any induction presupposes a prior abduction. A surprise or an inconsistency between the induction and the prior abduction provides the impetus for a new abduction. Such an abduction could be wholly novel or it could be, which is vastly more likely to be fruitful, a carefully selected variation on the original abduction.

At this point, Peirce's views on the boundaries between abduction and induction become fairly subtle; for he does not count every emendation of a hypothesis as a new abduction. For example, he counts it as an epistemic virtue of a hypothesis "that it may give a good 'leave,' as the billiard-players say" (EP 2:110, 1901). The idea is that the hypothesis contains some wiggle-room. We may think, for instance, that the data should be described by a quadratic relationship, and yet we may find it more instructive to fit a linear relationship because the residuals are more readily interpreted. The pattern of their deviations provides some information about how we might wish to adjust our hypothesis.

In starting with a simple hypothesis and successively modifying it within a class of possible hypotheses that are obvious from the start, we are, Peirce believes, engaged in induction, not abduction: "induction adds nothing. At the very most it corrects the value of a ratio or slightly modifies a hypothesis in a way which had already been contemplated as possible" (EP 2: 106, 1901). Peirce's idea seems to be that even a fairly specific hypothesis, such as that the data are quadratic, can be treated not as a precise claim but in a manner close to the original meaning of "paradigm," as an instance or exemplar that can serve as an index for a wider family of precise hypotheses. Thus, the quadratic hypothesis could be taken to be an index of the whole polynomial family; furthermore, just as enumerative induction of a statistical kind results in a narrowing of the bounds on the value that a true ratio could take, inductions of a nonstatistical kind can be taken as a narrowing of the subset of family members compatible with the truth. Idioscopic sciences, physics without a doubt, but possibly economics as well, are rapidly approaching a condition in which "we should no longer look for unexpected additions to our knowledge so much as to narrowing the limits between which it seems likely each truth will ultimately be found to lie" (R 678:28, 1910; quoted in the introduction to NEM III/1, p. xxvii, and in Eisele 1979: 371). The true abduction—the inference that might provide "unexpected additions to our knowledge"—is the replacement of one family of hypotheses by another.

The promise of both statistical and nonstatistical induction for Peirce is their property that, if we keep at it, they are bound in the fullness of time to reveal the falsity of our hypothesis, if it is indeed false—even if the falsity appears only beyond some particular limit of precision. There is no certainty of their success even in a negative sense: the falsifying observation may always lie just around the corner. And there is absolutely no certainty in the positive sense: although an observation may tell us that we are entertaining entirely the wrong family of hypotheses, it requires luck or *il lume natural* or a natural affinity for the truth—an affinity which can be grounded only in hope and not in knowledge—to pick the right family (CP 1.121, c.1896; EP 2: 107, 1901).

Peirce holds up Ricardo's theory of rent as a prime example of analytical reasoning—not only for economics, but for all science. The manner in which he moves seamlessly in his exposition of the theory from the specific details of rent to the question of tax incidence, which involves positing an independent causal role for demand—quite new since Ricardo—suggests that he regards the theory as a starting point, an initial template, an index of a family of hypotheses whose members differ in their scope and complexity, which is ripe for inductive precisification and which, in the manner of Kepler's introduction of the causal powers of the sun, can be joined to novel abductions to build an idioscopic economic model of real-world phenomena (CP 4.115, 1893).[19]

5. Ricardian Inference in the Harvard Lectures

Peirce's references to Ricardian inference are fleeting and undeveloped, even though he signaled his plans to elaborate on the topic. Based on the limited direct evidence, we have identified two apparently distinct possibilities for what Peirce understood the Ricardian inference to be: the primipostnumeral syllogism or analytical inference generally. And we have been left with a puzzle: on one interpretation, Ricardian inference is related to a sophisticated area of mathematics, while on the other interpretation, it is related to the methodology of empirical investigation; and we appear to have no compelling reason for choosing one interpretation over the other. But perhaps we do not have to choose. Yes, these interpretations are different; yet, we suggest they were not actually separate in Peirce's mind. He used the Harvard Lectures of 1903 concerning his view of pragmatism to braid together several threads related to infinity, continuity, abduction, and induction. And, although the terminology of Ricardian inference is not mentioned in the lectures, it is striking that early in the first lecture, Peirce draws on an economic model to illustrate the application of his pragmatic maxim (EP 2: 136–138). Peirce, we suggest, does not view the two interpretations as distinct but as aspects of a unified view of ampliative inference.

The deepest connection is found in Peirce's conception of mathematics. In his view, mathematics posits hypotheses and traces out their consequences without reference to facts. Yet, it is observational in the genuine but "very peculiar sense" that it constructs imaginary objects "according to abstract precepts, and then observes these imaginary objects, finding in them relations of parts not specified in the precept of construction" (CP 1.240, 1902). In the sixth Harvard Lecture, Peirce identifies these imaginary objects with *diagrams*:

> All necessary reasoning without exception is diagrammatic. That is, we construct an icon of our hypothetical state of things and proceed to observe it. This observation leads us to suspect that something is true, . . . and we proceed to inquire whether it is true or not. (EP 2: 212; see also CP 1.54, c.1896, and Ketner and Putnam 1992: 2–3)

Deductive inference thus follows exactly the same pattern as analytical inference: abduction (conjecture) followed by induction (inquiry) into truth. Deduction differs from ampliative inference only in the sense that we are the authors of the world being investigated—namely, of the diagram that serves as an icon of our hypothetical state of things.

In Section 3, we illustrated the primipostnumeral syllogism using one of Peirce's favorite examples: Zeno's supposed paradox of Achilles and the Tortoise. Peirce returns to their footrace at several points in the Harvard Lectures (EP 2:185–186, 210–211, 227, 236–237). Let us consider again Peirce's account of the "Achilles sophism" as an illustration of analytical inference applied to mathematics. Where do Zeno and subsequent paradox-mongers go wrong? For Peirce, an infinite multitude is not a collection of particulars; it has no reality other than its generality or its Thirdness, to use his metaphysical category elaborated in Lecture 3 (EP 2:179–195, especially 181–186). Its reality is characterized, as we saw in Section 3, by a generating relation that forms one of the premises of the Fermatian and the primipostnumeral syllogisms. The sophists construct a hypothetical world in which Achilles must occupy sequentially the spots previously occupied by the Tortoise. Their diagram of this hypothetical world is Fermatian, relying only on rational numbers, and the sequence, characterized by the generating relation, is denumerable. An inductive inquiry in such a world shows that the sophists are, in fact, correct: in their hypothetical world, Achilles cannot catch the Tortoise.

The problem with the paradox is not in its mathematics but in its logic. Ultimately, however, Peirce's point is not about the sophists' world. Pure mathematics may be hypothetical, but mathematics serves all sciences (CP 1.133, c.1894; CP 1.245, 1902). Mathematics is even more basic than logic, which itself is a servant of all other sciences. Mathematics "is purely hypothetical: it produces nothing but conditional propositions," whereas logic "is categorical" and "a normative science" (CP 4.240, 1902). Mathematics grips the real world through the iconic nature of its diagrams—that is, through analogy. But as we saw in Section 4, the abductive step in analytical inference is, for Peirce, a matter of constructing analogies or paradigms, while the inductive step is a matter of refining and checking the applicability of those analogies—in this case, checking the success of the sophists' generating relation in characterizing the relative positions of Achilles and the Tortoise in the real world. And Peirce declares it to be a failure.

We can easily observe that, in *perceptual* or *empirical* fact, Achilles occupies points that lie outside the set of countably infinite points characterized by the Fermatian generating relation. Where the paradox-mongers see the problem as a "contradiction of logic," Peirce sees them as having abandoned the most basic logical norms: "*Whose* logic? If it were mine that was in so flagrant violation of fact I would change it for one that died [*sic*] not lead from a true premise to a false conclusion" (NEM III/2: 803, 1903).[20] A generating relation based on a nondenumerable set (the primipostnumeral multitude), leads mathematically to Achilles catching the Tortoise, and that result is consistent with observations of the real world: it is, in this case, the better abduction.

In the Harvard Lectures, Peirce insists on the *reality* of the general and, in particular, of the features of the world that can be characterized only with respect to primipostnumeral generating relations. The lectures open with a restatement of his famous pragmatic maxim from 1878:

> Consider what effects that might conceivably have practical bearings we conceive the objects of our conception to have: then, our conception of those effects is the whole of our conceptions of the object. (EP 2: 135)

To give an example of the pragmatic maxim in action, Peirce asks, "*What is meant by saying that the probability of an event has a certain value, p?*" (EP 2: 136). To make the question of probability into a "practical" one, Peirce notes that "the great business of insurance depends on it" (ibid.).[21] He proceeds in the manner of analytical inference by constructing a paradigm case. Following the pole star of his economics, Ricardo, and his formalizer, Cournot, he treats the insurer as a profit-maximizing firm (see Wible 2014). The firm faces a downward-sloping demand curve, where the price of a policy (p) depends on the number of policies sold (n), which Peirce formalizes implicitly as

$$p = f(n), f' < 0. \tag{1}$$

The firm's problem is to maximize its profits defined as the value of policies sold (pn) less the amount paid out in insured losses (qln), which Peirce writes explicitly as

$$pn - qln \text{ or } (p - ql)n, \tag{2}$$

where q = the probability that an insurer will have to pay off on a particular policy and l = the value of the individual payoff.[22]

We can understand this model as the first (abductive) step in an analytical inference. But it also involves the primipostnumeral syllogism. The model defines a generating relation for p, and that relation depends on q, the probability. And what is probability? For Peirce, it is "a *statistical ratio* ... of the number of experiential occurrences of a specific kind to the number of experiential occurrences of a generic kind, in the long run" (EP 2: 137).

It would be easy to read this as a statement of a frequentist conception of probability, but that would require us to see his qualification "in the long run" as referring to some definite end point in the future. While not elaborating on the point in the Harvard Lectures, he rejects that interpretation of the long run.[23] Rather the "*long run* ... [is] an indefinitely long series of occurrences taken together in the order of their occurrence in possible experience" (EP 2: 138). While the set contains particular instances, the set itself can be characterized—just as any infinite set can be characterized, according to Peirce—only as a real general—that is, by its generating relation.

It would also be easy to read Peirce's definition of probability as restricting probabilities to rational numbers and therefore making the generating function Fermatian. That would simply be a species of the same genus of objections that Peirce raises to the frequentist conception of probability more generally. A sequence of rational numbers may have an irrational number as its limiting value, and the probability (q) is the limit of the statistical ratio *in the long run* (NEM II: 147, c.1895). The evidence for probability is particular, and it comes to us as concrete statistical facts and therefore expressible as rational numbers; but the probability itself is about possibility and must be expressible as a real number. For Peirce, such possibilities are quite real.

The mathematical solution to the insurance problem would be obvious to any student of Cournot (or by 1903, more likely of Alfred Marshall): take the derivative with respect to n of the expression for profit in (2), set it equal to zero, and solve for p. This would, in fact, be an application of the primipostnumeral syllogism. And Peirce gives the solution:[24]

$$p = ql - \frac{\partial p}{\partial n} n \, . \tag{3}$$

But Peirce does not refer to this straightforward application of the differential calculus in the lecture. One explanation is that he was trying to respect William James' request that he not tax the mathematical abilities of his audience.[25] But a more interesting—and, we think, more compelling possibility—is that Peirce wanted to relate his illustration of the pragmatic meaning of probability to his account of abduction. After all, he says in Lecture VII that "the question of pragmatism ... is nothing else than the question of the logic of abduction" (EP 2: 234).

His explanation of the optimal price argues more in the style of Ricardo than of Cournot. Ricardo, in his chapter on rent, illustrates the determination of rent for any parcel of land by comparing it to a discrete sequence of parcels ordered from high productivity to low productivity. If productivity declined continuously, then the "the last land ... cultivated ... will ... pay no rent" and any land enjoying a higher productivity than this marginal land will pay the entire differential as rent to the landlord (Ricardo 1821 [1951], ch. 2). Ricardo's account of rent can also be derived using the differential calculus and invoking the primipostnumeral syllogism.

Peirce's explanation starts, as does Ricardo's explanation of rent, with the assumption that the insurer, like the farmer (who is a capitalist and not, like the landlord, a rentier), has successfully maximized his profits. He then considers how those profits would change if the insurer sold one more or one fewer policy. While he writes $\frac{\partial p}{\partial n}$ as a derivative, he *defines* it in this passage discretely as "the amount by which the price would have to be lowered in order to sell one policy more" (EP 2: 136). Starting with the expression for profits in (2), he computes their values for $n + 1$ and $n - 1$, and shows that these imply lower and upper bounds for the optimal price at n: [26]

$$ql \quad \frac{\partial p}{\partial n}n+\frac{\partial p}{\partial n}<p<ql-\frac{\partial p}{\partial n}n-\frac{\partial p}{\partial n}\cdot \tag{4}$$

He then concludes that "since . . . $\frac{\partial p}{\partial n}$ is very small, it must be very close to the truth to write $p=ql-\frac{\partial p}{\partial n}n$" (EP 2: 137). In fact, the argument, as Peirce knows, is not complete, but depends on an additional assumption—namely, that $\frac{\partial p}{\partial n}$ measured as the change in price to sell one more policy takes the same value on both sides of equation (4), which requires that "very small" be actually *infinitesimal* and that the demand curve in (1) and its slope be continuous.

The bounds in inequality (4) can be estimated and precisified inductively using statistics to determine estimates of the probability (q) and the shape and location of the demand curve. Practical insurance companies would devote resources to such inductive investigation. But notice that Peirce does not argue that the price of the policy (p) is indeterminate between the bounds. Rather it is the optimum price in the long run, which means, as it did for probability, that the firm looks forward to an indefinite extension of its business (see also NEM II:48, c.1895). This long run view requires treating the model as providing a generating relation for an infinite sequence in which the true price, although it may be estimated by data expressed in rational numbers, is in itself a real number and requires a primipostnumeral and not a Fermatian analysis.

Induction and statistical precisification do not reach an end point in any finite time; we can never reach a collection of evidence that will tells us that we have precisely identified the probability. And yet, Peirce argues, the probability—and, by extension, the price that depends on it—"must be of the nature of a *real fact* and not a mere *state of mind*. For facts only enter into the solution of the problem of insurance" (EP 2: 137). The pragmatic element for Peirce is that these real facts govern the future and that our inferential problem is how to align our understanding with the constraints that they impose. The statistics that enter into the estimation of the probability and the demand curve are past, particular facts. But our interest is not in describing the past or characterizing it efficiently, but rather in knowing how to guide our expectations and actions. Thus, in Lecture VII, Peirce rejects the view, which he associates with Comte among others, that "we are to believe only what we see" and "that it is unscientific to make predictions,—unscientific, therefore, to expect anything" (EP 2: 236). Rather, science and practical action require us to try to characterize the general, which governs the future and which warrants prediction.

When the insurance company posits a role for probability in determining the optimal price, it makes an abduction. Induction may produce more precise values for that probability, but it also opens the abduction to possible refutation. Immediately after discussing the insurance model in Lecture I, Peirce points out that not every stochastic process converges to a well-defined probability. And "probabilities 1 and 0 are very far from corresponding to certainty *pro* and *con*" (EP 2: 139). Peirce apparently refers to outcomes that in modern measure theory are referred to as having Lebesgue measure zero: an infinite number of disconnected elements can be removed from an infinite set without changing the probability measure of that set, even when that measure is one or zero. For Peirce, the consequences of these two points are, first, that the attribution of probability in the world is always an abductive conjecture and subject to inductive falsification, and second, that the falsifier is never just the collection of particular facts, but the general relationship that governs those facts. The Achilles sophism is rejected not simply by the facts of the sequence of relative positions of Achilles and the Tortoise, but by the inconsistency of those facts with a description of space as discrete and the failure to find an inconsistency with a description of space as continuous. Similarly, the insurance model could be rejected, not by simply finding one-off cases of surprising losses, but by those losses forming a pattern that was inconsistent with the model. This is the basis for statistical testing, an area in which Peirce was himself a pioneer (see Hacking 1990: 210).

6. The Logic of Economics is the Logic of Science

The two interpretations of Ricardian inference are thus joined at the hip. Analytical inference involves an abductive step, which in Peirce's understanding always refers to the general and to open possibilities, which require the analysis of infinite sets. Infinities may be discrete and countable, the realm of rational numbers and the Fermatian syllogism, or continuous and uncountable, the realm of the real numbers and the primipostnumeral syllogism. Reasoning with either syllogism follows the same pattern as analytical inference: abduction, a matter of setting up a diagram in the form the generating relation, and inductively testing supposed consequences by observing the diagram. Such reasoning differs from empirical analytical inference only in that the constraints on what we observe are built into the hypothetical premises, whereas in the empirical case, the real world itself imposes the constraints, whatever we may ourselves suppose. The utility of models, of diagrams, of mathematics for understanding the world arises from our ability to construct analogies between the world of the model and the world that generates the facts.

David Ricardo was a stockbroker, member of Parliament, and the owner of extensive agricultural properties. His economic analysis focused on the development of the underlying theory of the economy. He was a modeler *avant la lettre*, and he has frequently been criticized for having been captured by his models. But for Peirce, this demonstrated that, even though Ricardo lacked mathematical form, he had a mathematical mind. His mathematics implicitly used the primipostnumeral syllogism. And his application of the syllogism to economics was an empirical application of analytical inference in exactly the same sense that the analysis of Achilles and the Tortoise was empirical. Only common observations known to anyone with experience in the world were needed to decisively discriminate inductively between the abduction of the paradox-mongers and Peirce's own abduction with respect to Achilles and the Tortoise. Similarly, Ricardo was no less empirical for testing his abductions about income distribution (rent, profits, wages) against a lifetime's informal observations of the real economy.

Fermat and Cantor were ranked high in Peirce's pantheon of mathematicians. Similarly, Ricardo and Cournot stood high in Peirce's pantheon of social scientists. His term "Ricardian inference" captures the close connection between hypothetical mathematical reasoning, exemplified in the primipostnumeral syllogism, and analytical inference displayed in Ricardo's awkward (and Cournot's elegant) applications of reasoning using infinitesimals to understand the economy. To those who know Peirce as a natural scientist, it is perhaps surprising that he turned to economics to illustrate a logical point of such wide applicability. It reflects, however, Peirce's exceptional intellectual range and his catholic conception of the scope of science and logic.

Appendix: The Primipostnumeral Syllogism and Ricardo's Theory of Rent

Peirce is clear that he intends the primipostnumeral syllogism to analogize to the Fermatian syllogism (CP 4.207–209, 1897).[27] Peirce refers to primipostnumeral *collections*, which we may regard as sets with the cardinality of the real numbers. "In like manner [to the Fermatian syllogism], in order to prove that anything is true of a primipostnumeral collection ... we must consider that collection in its primal arrangement or with reference to a relation equivalent to that of its primal arrangement" (CP 4.209). That is, there must be a generating relation that imposes a structure or ordering on the units that make up the collection. Peirce's statement of the primipostnumeral syllogism involves his concept of *packs*. We first state the general form of the syllogism (see CP 4.209) in the form that parallels the example of the syllogism of Achilles and the Tortoise in Section 3 of the main text and then explain the various terms:

A.i)	the units of the collection ($\prod$s) are ordered according to a relation p and the packs (Ps) that are "parts of the collection" [subsets] according to a relation s;	(generating relations)
A.ii)	$\prod_0$, the unit of P_0, is X;	(particular property)
A.iii)	if every $\prod$ of any pack is X, then every $\prod$ of the pack which is s to that pack is X;	(conditional property)
∴ A.iv)	every $\prod$ is X.	[Q.E.D.]

Here X is a property or predicate, such as "is to the left of" or "is less than." Peirce develops his concept of packs formally in CP 4.207. The details are not easy to follow, and we believe that some of his definitions may not be watertight and that there may be some notational mistakes. This is not surprising in a manuscript that was never edited for publication. However, the main thrust seems to be this: packs are disjoint subsets of the collection. Perhaps, Peirce thinks of the units as cards and the packs as dividing the cards into subsets or decks. Any individual card can be included in only one deck. Peirce states twelve conditions that define the relationship between the units of the collection and the packs. He claims that any collection that fulfills these conditions is primipostnumeral. He further claims that the pack P_0, which in the primipostnumeral syllogism is the first pack in the primal order or generating relation, contains only one unit ($\prod_0$) and that subsequent packs contain double the number of units of the preceding pack.

Although Peirce asserts that the primipostnumeral syllogism was used (implicitly) by Ricardo in his theory of rent, he does not state explicitly how he would formulate Ricardo's argument in a syllogistic form. Here we offer our own conjecture of how such an argument might go. Ricardo (1821 [1951], ch. 2) conceives of the agricultural economy as divided into landlords, who own the land and are paid rent; farmers, who in return for profits lease the landlord's property and use monetary capital to apply physical resources and hired labor to generate valuable crops; and workers, who earn wages. A critical assumption is that the structure of the market is such that, owing to competition and the interaction of supply and demand, all workers earn the same wages and all farmers (capitalists) earn the same percentage return on capital outlays. Ricardo assumes that land comes in different qualities measured in terms of its productivity per unit of applied capital and that even the same unit of land produces greater output as more capital is directed to it, but at a decreasing rate (what economists today call *diminishing returns* to factors of production). Ricardo frequently uses numerical examples in which a few discrete units of land display discrete differences in productivity. But that is an expository device, and he is clear that capital, which is effectively infinitely divisible, can be applied and faces *continuously* decreasing returns.

Beyond these numerical examples, Ricardo himself used no mathematics. But his argument is easily captured in a simple mathematical model in which the land of the entire economy, which is used to produce corn, is brought into production continuously. The active parties are the farmers, who seek to maximize their profits, defined as

$$\varphi = f(L) - kL,$$

where L = subset of available land actually farmed, and k = the capital outlay per unit of land, which is assumed to be constant.[28] The function $f(.)$ is the value of the output of corn per incremental unit of L. Ricardo conceives of the land as ordered from most productive to least productive and to be brought into production in that order. We can thus think of $f(L)$ as a curve on a graph with L on the horizontal axis and output per incremental unit on the vertical axis. The curve is downward-sloping, taking a high value on the left and a low (possibly zero value) on the right.

To maximize profits, farmers simply follow the rule: extend production to all units of land (L_i) for which $\varphi_i \geq 0$. Since costs per unit of land are constant, rent on the *i*th unit of land (R_i) can be defined as

$$Ri = f(L_i) - k;$$

that is, once the farmer has earned the economy-wide rate of return on his outlay of capital, the value of the remainder of the corn goes to the landlord.

It is worth reminding ourselves that because the cardinality of L is primipostnumeral, the units here are infinitesimal. It is central to Peirce's view of infinity that, while many mathematical arguments can be made in terms of limits, this is often clumsy and the infinitesimals are legitimate. In the text in which the primipostnumeral syllogism is introduced, he provides an argument that he claims shows that "every unit of a primipostnumeral collection admits of being individually designated and exactly described in such terms as to distinguish it from every other unit of the collection" (CP 4.211, 1897). That is to say, we can legitimately discuss a set with a single infinitesimal as its member, and we do not need to explain them away through the device of the limits of sequences of ratios.

In terms of the syllogism template A.i–A.iii, Ricardo's collection is the continuous quantity of land, so that the L_i's are Peirce's Πs. The property X in this case is "is worth farming" (i.e., "is land such that $\varphi_i \geq 0$"). Define "$\prec$" to mean "is ordered ahead of" (or in terms of a graph "is to the left of"). We can now state Ricardo's argument in the form of a primipostnumeral syllogism. We begin with a collection Λ consisting of all L_q's such that $f(L_q)$ is greater than $f(L_0)$, where L_0 is that unit for which $f(L_i) - k = 0$—in Ricardo's terms, L_0 is "the land that pays no rent" and it lies somewhere to the right on the downward-sloping corn-productivity curve. The primipostnumeral syllogism then is:

A.i′)	a) [the relation p]: the units of the collection Λ are such that, for any distinct units L_i and L_j, $L_j \prec L_i$ if, and only if, $f(L_j) > f(L_i)$; b) [the relation s]: for any pack P_k of the collection Λ, let L^*_k be the first unit (i.e., that unit for which $f(L_k)$ is the minimum among the units in the pack); then, for any distinct packs, P_m and P_n, $P_m \prec P_n$ if, and only if, $f(L^*_m) > f(L^*_n)$.	(generating relations)
A.ii′)	the pack P_0 contains only the unit L_0, and note that L_0 is worth farming;	(particular property)
A.iii′)	consider any two packs P_m and P_n, such that $P_m \prec P_n$, for which every $L_j \in P_n$ is worth farming, then every $L_i \in P_m$ is worth farming;	(conditional property)
∴ A.iv′)	every $L_q \in \Lambda$ is worth farming.	Q.E.D.

Premise A.ii′ follows directly from the definition of Λ. Premise A.iii′ is justified as follows: for every L_j in P_n, $L_j \prec L^*_{\mathrm{n}}$ and $L^*_m \prec L^*_{\mathrm{n}}$; so since L^*_{n} is worth farming and L^*_m by the generating relation (p) has a higher incremental rate of output for the same costs, it too must be worth farming; and similarly, since every other L_i in P_m has a higher incremental rate of output than L^*_m, they too must be worth farming. Whether the conclusion A.iv′ follows depends on whether the primipostnumeral syllogism is, in fact, a valid inferential form. Peirce clearly thought so. We withhold judgment.

This application of the primipostnumeral syllogism makes rather heavy weather of an argument that Ricardo presents convincingly with a few informal examples. Equally, economists after Cournot would simply use the differential calculus to reach the same conclusion in one line. Peirce's goal, however, is not to present a practical mode of calculation but to expose the set-theoretic structure underlying the calculus when the notions of nondenumerable infinites and real infinitesimals are taken seriously and to show that structuring the calculus around limits is unnecessary and constraining. He is justified, in his own mind, for invoking Ricardo, because Ricardo constructs his argument without differentiation by arguing from the borderline case ("the land that pays no rent") to the final conclusion in a pattern that parallels the primipostnumeral syllogism.

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REFERENCES

Bentham, Jeremy (1816). *Chrestomatia*. London: Payne and Foss; and R. Hunter.

———. (1952). *Jeremy Bentham's Economic Writings: Critical Edition Based on his Printed Works and Unprinted Manuscripts*, vol. I. W. Stark, editor. London: George Allen & Unwin.

Blaug, Mark (1997). *Economic Theory in Retrospect*, 5th edition. Cambridge: Cambridge University Press.

Brent, Joseph (1998). *Charles Sanders Peirce: A Life*, revised and enlarged edition. Bloomington: Indiana University Press.

Burks, Arthur W. (1946). "Peirce's Theory of Abduction," *Philosophy of Science* 13(4), 301–306.

Cantor, Georg (1874). "Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen," *Journal für die Reine und Angewandte Mathematik* 1874(77), 258–262.

[Century Dictionary] (1889). *The Century Dictionary and Cyclopedia: An Encyclopedic Lexicon of the English Language*. New York: Century.

Cherriman, John Bradford (1857). "Review of Cournot," *Canadian Journal of Industry, Science and Art* N.S. 2, 185–194.
Cohen, Paul J. (1963). "The Independence of the Continuum Hypothesis," *Proceedings of the National Academy of Sciences of the United States of America* 50(6), 1143–1148.
———. (1966). *Set Theory and the Continuum Hypothesis.* Reading, MA: W.A. Benjamin.
Comte, Auguste (1830[1905]). *The Fundamental Principles of the Positive Philosophy: Being the First Two Chapters of "Cours De Philosophie Positive*," Paul Descours and H. Gordon Jones, translators. London: Watts.
Cournot, Antoine Augustin (1838[1927]). *Recherches sur les Principes Mathematiques de la Theorie des Richesses.* Paris: Hachette. All quotations are of the English translation: *Researches into the Mathematical Principles of the Theory of Wealth*, Nathaniel T. Bacon, translator. New York: Macmillan.
Dauben, Joseph W. (1982). "Peirce's Place in Mathematics," *Historia Mathematica* 9, 311–325.
de Fontenay, R. (1864). "Principes Mathematiques de la Théorie des Richesses par M. Cournot," *Journal des économistes* 11, 231–251.
Dimand, Robert W. (1995). "Cournot, Bertrand, and Cherriman," *History of Political Economy* 27(3), 563–78.
Douven, Igor (2017a). "Abduction," *Stanford Encyclopedia of Philosophy* (Summer 2017 edition), Edward N. Zalta, editor, URL = https://plato.stanford.edu/archives/sum2017/entries/abduction/
———. (2017b). "Peirce on Abduction," *Stanford Encyclopedia of Philosophy* (Summer 2017 edition), Edward N. Zalta, editor, URL = https://plato.stanford.edu/archives/sum2017/entries/abduction/peirce.html
Eisele, Carolyn (1979). *Studies in the Scientific and Mathematical Philosophy of Charles S. Peirce.* The Hague: Mouton.
Eltis, Walter (1984). *The Classical Theory of Economic Growth*. London: Macmillan.
Fann, K. T. (1970). *Abduction*. The Hague: Martinus Nijhoff.
Frankfurt, Harry G. (1958). "Peirce's Notion of Abduction," *The Journal of Philosophy* 55(14), 593–597.
Gödel, Kurt (1940). *The Consistency of the Continuum Hypothesis*. Princeton University Press.
Hacking, Ian (1990). *The Taming of Chance*. Cambridge: Cambridge University Press.
Heron, Timothy (1997). "C. S. Peirce's Theories of Infinitesimals," *Transactions of the Charles S. Peirce Society* 33(3), 590–645.
Hoover, Kevin D. (1994). "Pragmatism, Pragmaticism, and Economic Theory," *New Perspectives on Economic Methodology*, Roger Backhouse, editor. London: Routledge, pp. 286–315.
Humphrey, Thomas M. (2010). "Marshallian Cross Diagrams," in Mark Blaug and Peter Lloyd, editors, *Famous Figures and Diagrams in Economics*. Cheltenham: Edward Elgar, pp. 29–37.
James, William (1902–1905[2002]). *The Correspondence of William James*, vol. 10, I. K. Skrupskelis and E. M. Berkeley, editors. Charlottesville, VA: University of Virginia Press.

Jevons, William Stanley (1879). *The Theory of Political Economy*, 2nd edition. London: Macmillan.

———. (1965). *The Theory of Political Economy*, 5th edition. New York: Augustus M. Kelley, reprint of 5th edition of 1957.

Josephson, J. R., and S. G. Josephson (1994). *Abductive Inference: Computation, Philosophy, Technology*. Cambridge: Cambridge University Press.

Ketner, Kenneth, and Hilary Putnam (1992). "Introduction" to Peirce, *Reasoning and the Logic of Things: The Cambridge Conferences Lectures of 1898*, Ketner and Putman, editors. Cambridge, MA: Harvard University Press.

Keynes, John Neville (1917). *The Scope and Method of Political Economy*, 4th ed. London: Macmillan.

Kuhn, Thomas (1962). *The Structure of Scientific Revolutions.* Chicago: Chicago University Press.

Lipton, Peter (2004). *Inference to the Best Explanation.* London: Routledge, 2004.

Mabsout, Ramzi (2015). "Abduction and Economics: The Contributions of Charles Peirce and Herbert Simon," *Journal of Economic Methodology* 22(4), 1–26.

Moore, Matthew E., ed. (2010a). *New Essays on Peirce's Mathematical Philosophy.* Bloomington: Indiana University Press.

Moore, Matthew E. (2010b). "Peirce's Cantor," in Moore (2010a), pp. 323–362.

Morgan, Mary S. (2012). *The World in the Model: How Economists Think.* Cambridge: Cambridge University Press.

Murphey, Murray G. (1993). *The Development of Peirce's Philosophy*. Indianapolis: Hackett.

Parker, Kelly A. (1998). *The Continuity of Peirce's Thought.* Nashville: Vanderbilt University Press.

Peirce, Benjamin (1859a). "[Letter] To Messr. Habicht and Holbrooke of the *American Agency of the International Life Assurance Society*," in *Massachusetts Reports on Life Insurance: 1859–1865,* by the Insurance Commissioners. Boston: Wright and Potter, State Printers, pp. 96–98.

———. (1859b). "The International Life Assurance Society. *Card from Professor Peirce*," in *Massachusetts Reports on Life Insurance: 1859–1865,* by the Insurance Commissioners. Boston: Wright and Potter, State Printers, pp. 105–106.

Peirce, Charles S. (1931–1958). *The Collected Papers of Charles Sanders Peirce*, vols. 1–8. Charles Hartshorne, Paul Weiss, and Arthur Burks, editors. Cambridge, MA: Harvard University Press.

———. (1976). *The New Elements of Mathematics by Charles S. Peirce*, vol. I–IV. Carolyn Eisele, editor. The Hague: Mouton.

———. (1982–2010). *Writings of Charles S. Peirce: A Chronological Edition*, vols. 1–6 and 8. Max H. Fisch, Edward C. Moore, Christian J. W. Kloesel et al., editors. Bloomington: Indiana University Press.

———. (1992–1998). *The Essential Peirce*, vol.1–2. Nathan Houser, Christian Kloesel, and the Peirce Edition Project, editors. Bloomington: Indiana University Press.

Pietarinen, Ahti-Veikko (2006). "Interdisciplinarity and Peirce's Classification of the Sciences," *Perspectives on Science* 14(2), 127–152.

———. (2010). "Which Philosophy of Mathematics is Pragmaticism?," in Moore (2010a), pp. 59–79.

Pietarinen, Ahti-Veikko, and Francesco Bellucci (2014). "New Light on Peirce's Conceptions of Retroduction, Deduction, and Scientific Reasoning," *International Studies in the Philosophy of Science* 28(4), 353–373 (http://dx.doi.org/10.1080/02698595.2014.979667).
Popper, Karl (1963). *Conjectures and Refutations: The Growth of Scientific Knowledge.* London: Routledge & Kegan Paul.
Rescher, Nicholas (1978a). *Peirce's Philosophy of Science*. Notre Dame: University of Notre Dame Press.
———. (1978b). *Scientific Progress: A Philosophical Essay on the Economics of Research in Natural Science.* Oxford: Basil Blackwell.
———. (1989). *Cognitive Economy: The Economic Dimension of the Theory of Knowledge.* Pittsburgh: University of Pittsburgh Press.
Ricardo, David (1821[1951]). *On the Principles of Political Economy and Taxation*, volume 1 of Piero Sraffa, editor. *The Works and Correspondence of David Ricardo.* Cambridge: Cambridge University Press.
Robbins, Lionel (1935). *An Essay on the Nature and Significance of Economic Science.* London: Macmillan.
———. (1998). *A History of Economic Thought: The LSE Lectures*. Steven G. Medema and Warren J. Samuels, editors. Princeton: Princeton University Press.
Robin, Richard. (1967). *Annotated Catalogue of the Papers of Charles S. Peirce*. Amherst: University of Massachusetts Press.
Robinson, Abraham (1966). *Non-standard Analysis*. Amsterdam: North-Holland.
Schumpeter, Joseph A. (1954). *History of Economic Analysis*. London: George Allen & Unwin.
Töben, Gerald Fredrick (1986). *Peirce's Fallibilism and Popper's Falsification Theory.* Peace Books.
Tuzet, Giovanni (2018). "On 'The Economy of Research'," *Transactions of the Charles S. Peirce Society* 54(2), 129–133.
Voelkel, James R. (2001). *The Composition of Kepler's* Astronomia Nova. Princeton: Princeton University Press.
Walton, Douglas (2004). *Abductive Reasoning*. Tuscaloosa: University of Alabama Press.
Wible, James R. (1998). *The Economics of Science: Methodology and Epistemology as if Economics Really Mattered*. London: Routledge.
———. (2014). "Peirce's Economic Model in The First Harvard Lecture on Pragmatism," *Transactions of the Charles S. Peirce Society* 50(4), 548–580.
———. (2018). "Game Theory, Abduction, and the Economy of Research: C. S. Peirce's Conception of Humanity's Most Economic Resource," *Transactions of the Charles S. Peirce Society* 54(2), 134–161.
———. (2020a). "C. S. Peirce's Theory of Abductive Expectations," *European Journal of the History of Economic Thought* 27(1), 2–44.
———. (2020b). "Why Economics is an Evolutionary, Mathematical Science: How Could Veblen's View of Economics Been So Different than C. S. Peirce's?, *Journal of the History of Economic Thought*, forthcoming.
Wible, James R., and Kevin D. Hoover (2015). "Mathematical Economics Comes to America: Charles S. Peirce's Engagement with Cournot's *Recherches sur les*

Principes Mathematiques de la Théorie des Richesses," Journal of the History of Economic Thought 37(4), 511–536.

———. (2020). "The Economics of Trade Liberalization: Charles S. Peirce and the Spanish Treaty of 1884," *European Journal of the History of Economic Thought*.

NOTES

1 We are currently writing a book on Peirce's engagement with economics. One chapter of that book will consider Peirce's paper on the economy of research in detail, while a companion paper to this one addresses his analysis of the Spanish Treaty (Wible and Hoover 2020). Following standard conventions among Peirce scholars, references to Peirce's *Collected Papers* are generally indicated as "CP volume number.paragraph number." References to larger divisions or to parts of the text outside of the numbered paragraphs are indicated explicitly (e.g., "CP 1, ch. 4" or "CP 8, p. 283"). References to the *Writings of Charles S Peirce* are given as "W volume number: page number." References to *The New Elements of Mathematics* are given as "NEM volume number: page number." References to Peirce's manuscripts in the Houghton Library of Harvard University are given as "R manuscript number: page number," using the manuscript numbers provided in Robin 1967.

2 The limited literature on Peirce and economics includes Rescher (1978a, 1978b, and 1989), Hoover (1994), Wible and Hoover (2015), and Wible (2018, 2020a, and 2020b).

3 The economy of research was the focus of a workshop at Bocconi University (Milan) on October 5–6, 2017. Papers from that workshop are published in the spring 2018 issue number of the *Transactions of the Charles S. Peirce Society* (v.54, n.2). See Tuzet (2018) for an overview.

4 On the marginal revolution, see Schumpeter (1954), ch. 5, or Blaug (1997), ch. 8.

5 Wible and Hoover (2015) provide a detailed account of the core of Cournot's analysis and Peirce's reaction to it.

6 Ricardo also recognizes the independent discovery of the theory in a pamphlet published in 1815 by an anonymous "Fellow of University College, Oxford," who is now known to be Edward West.

7 Following Ricardo, Peirce recognizes two distinct, but logically similar margins: what modern economists refer to as the *intensive margin*—that is, to the increasingly smaller additions to output that result from the increasing use of inputs (labor, fertilizer, etc.) and the *extensive margin*—that is, to the increasingly smaller output that arises from bringing intrinsically and increasingly worse land into production (CP 4.115, 1893; Ricardo 1821[1951]: 70–72).

8 The fragmentary works in which economics is mentioned include the "Sketch," a syllabus for sixty lectures on logic intended for, but apparently never delivered as, a course at the Johns Hopkins University in 1883 (W 4: 476–489, especially p. 489; for context, see W 4:lx), in which economics would have been discussed between lectures on the kinetic theory of gases and anthropomorphic science; "Qualitative Logic" (W 5: 323–371, 1886), for which the section on economic inference was never completed; a prospectus for a twelve-volume work

on the principles of philosophy (CP 8: 282–286; esp: 285; c,1893), in which mathematical economics would be covered in volume VIII, titled *Continuity in the Psychological and Moral Sciences*, and Walras and Marshall were to be deployed as part of a proof of free will (volume VII would address "evolutionary chemistry" and volume IX a mathematical approach to comparative biography).

9 Fermat's last theorem—really his conjecture, since it is believed that, despite a marginal note to the contrary, he did not have a correct proof—states that, for any positive integers a, b, and c, there is no integer $n > 2$ such that $a^n + b^n = c^n$. It was finally proved in 1995 by Andrew Wiles, using mathematical techniques unknown to Fermat.

10 Fermat himself referred to the method as "indefinite descent" (CP 4.110, 1893). A very simple example of the Fermatian inference: *Theorem*: For all $n \geq 1$, $3^{n+1} - 1$ is even. *Proof*: i) Let $n = n_0 = 1$, then $3^{n_0+1}-1=3^{1+1}-1=3^2-1=8$, which is even. ii) Suppose that the conjecture holds for some element $k > 1$, so that 3^k-1 is even. Consider element $k + 1$, $3^{k+1}-1$, which can be written $3^{k+1}-1 = 3(3^k-1)+2$. The term in parentheses is even by assumption, and any integer multiple of an even number is even, and adding any even number to an even number results in an even number. Thus, iii) *if* element k is even, then element $k + 1$ is even; and for any k whatsoever, a chain can be constructed from $n = 1$ to $n = k$ in which the successive elements of the series can be shown to be even. *Q.E.D.*

11 "I pay full homage to Cantor. He is indisputably the *Hauptförderer* [chief promoter] of the mathematico-logical doctrine of numbers" (CP 4.331, c.1905). Peirce had been interested in conceptions of infinity as early as 1881, before he had first read any of Cantor's papers, and he claimed to have begun a study of number theory and "pushed them to considerable results before Cantor took up the subject" (NEM III 1: 333, 1903; see also CP 3.252–288, 1881). Dauben (1982) discusses Peirce's 1881 paper "On the Logic of Number," its relation to Cantor and Dedekind and to conceptions of infinite sets and infinitesimals. Of Dedekind, Peirce writes: "his little book *Was sind und was sollen die Zahlen?* is most ingenious and excellent. But it proves no difficult theorem that I had not proved or published years before, and my paper had been sent to him" (CP 4.331, c.1905). Dedekind's claims of priority clearly rankled Peirce. Peirce's philosophy of mathematics is now the subject of a collection of essays, Moore (2010a). Also, Moore (2010b) specifically considers Peirce's interest in Cantor. Pietarinen (2010) raises the more general question of how Peirce's ideas on mathematics fit with other philosophies of mathematics.

12 The conjecture that the real numbers are $\aleph_1$ is known as the *continuum hypothesis*. Like Cantor, Peirce presumed that it was true. In fact, its truth remains an open question. Gödel (1940) showed that the continuum hypothesis could not be disproved within Zermelo-Frankel set theory supplemented by the axiom of choice. Cohen (1963) showed that it could not be proved in the same context. Thus, it is independent of Zermelo-Frankel set theory plus the axiom of choice. Both Gödel's and Cohen's proofs take the consistency of Zermelo-Frankel set theory as given, but though this is widely believed, it too has not been proven. (See Cohen 1966 for a further discussion.) The real numbers are, therefore, known to have a higher cardinality than the integers; yet they may not have the cardinality $\aleph_1$.

13 Eisele (1979: 165) claims that "Peirce was one of the last stalwarts to defend the use of the infinitesimal in the structure of the calculus" (see also Eisele

1979: 101, and Heron 1997). Murphey (1993: 119–120), originally published in 1961, notes that modern mathematicians reject infinitesimals in favor of limits. Nonetheless, just a few years later, Peirce's claims for the consistency and tractability of infinitesimals would seem to be vindicated by what is now termed "nonstandard analysis." Robinson (1966) demonstrated the logical consistency of a mathematics that includes infinitesimals (see Eisele 1979: 246–248; Parker 1998: 93–99, especially pp. 98–99). Putnam writes: "Peirce's own theory of the continuum differs from Robinson's 'nonstandard' construction (although we believe it also could be shown to be fully consistent, relative to our present-day set theory" (in Ketner and Putnam 1992: 90). Ketner and Putnam (1992: 37–54) provide a detailed discussion of Peirce's ideas of the continuum and infinitesimals.

14 Weighing into contemporary debates among political economists over *Historismus* versus *a priori* methods (and anticipating debates between Institutionalists and neoclassical economists), Peirce goes on: the Analytical Method "is reprobated by the whole Hegelian army, who think it ought to be replaced by the 'Historic Method,' which studies complex problems in all their complexity, but which cannot boast any distinguished successes" (CP 1.64; see Keynes 1917: 314–327; Robbins 1935: 79–83, 1998, Lectures 25 and 26; Schumpeter 1954, ch. 4, part 2).

15 These criteria, along with other aspects of ampliative reasoning, are set out extensively in 1883's "A Theory of Probable Inference" (W 4: 408–450, especially 410–413 and 427–433). See also CP 1.67 and 93, c.1896; CP 2.775 and 788–790, 1901; CP 7.209–211, EP 2: 98–101, 1901.

16 Notable early explorations of Peirce's conception of abduction are Burks 1946, Frankfurt 1958, and Frankfurt 1970. Later ones include Josephson and Josephson 1994, Walton 1994, Douven 2017a and 2017b, and Pietarinen and Bellucci 2014. Josephson and Josephson (1994, pp. 2–3) note the inspiration Peirce has had on research in artificial intelligence and computation and that an important algorithm for medical diagnosis was named "PEIRCE." Peirce's conception of abduction has been applied to economic topics by Masbout (2015) and Wible (2020a).

17 On the history of models in economics, see Morgan 2012.

18 Perhaps because he counted astronomy among the many sciences to which he made important contributions, Peirce frequently illustrated methodological points with reference to Kepler (W 4: 419, 1883; CP 7.419, c.1893; CP 1.70–79, c.1986; EP 2: 83, 1901; CP 2.96–97, 1902). Peirce's reading of Kepler's accomplishments comports well with modern scholarship. Mill, in Peirce's view, misread Kepler, denying "that there was any reasoning in Kepler's procedure" and characterizing it instead as "merely a description of the facts" (CP 1.71, c.1896). In a detailed examination of Kepler's work, Voelkel (2001) reaches the same conclusions as Peirce for essentially the same reasons. Voelkel explains that astronomy in Kepler's day referred to the mathematics of celestial observations—that is, to the mere description of the facts—and it was thought to have achieved its ends when it could offer an accurate systematic description of the movements of the heavenly bodies (p. 24). In contrast, cosmography concerned itself with the nature of those bodies and was a branch of *physica* or even metaphysics, as the term was used at the time (p. 26 ff.). Thus, astronomers had no problem with entertaining Copernicus's heliocentric model, so long as it was regarded merely

as a way of generating the patterns of observation, while cosmographers, who were concerned about the real composition of the heavens, frequently rejected it as contrary to scripture. Voelkel, like Peirce, reads Kepler as radically rejecting the division between astronomy and cosmography or *physica* in insisting on a causal explanation not only to achieve the ends of cosmography but to perfect astronomy (*inter alia* pp. 28, 37, 127). Voelkel also notes the manner in which Kepler modeled the astronomical data iteratively in the manner that Peirce emphasizes (p. 205).

19 Peirce's treatment of Ricardo is deeply consistent with Morgan's (2012, ch. 2), not only as a generic "modeler" but as one who used the generic model as a template for quantitative refinement. Both Peirce and Morgan stand in stark contrast to the interpretive tradition exemplified by Schumpeter's (1954: 473) coinage of the term "Ricardian vice." Where Peirce and Morgan see Ricardo as fundamentally an *analytical* economist—that is, as an economic theorist who provides a basis for quantification and inductive precisification, Schumpeter sees him as "not a mind that is primarily interested in either fundamentals or wide generalizations.… His interest was in the clear-cut result of direct, practical significance" (p. 472). His method was to pile "one simplifying assumption upon another until, having really settled everything by these assumptions, he was left with only a few aggregative variables between which, given these assumptions, he set up simple one-way relations so that, in the end, the desired results emerged almost as tautologies" (pp. 472–473). The *Ricardian vice* refers to drawing solutions to practical problems from such reasoning, and Schumpeter likens it to "implicit theorizing"—a practice that Leontief attributed to, and criticized in the work of, John Maynard Keynes, whom he sees as a similarly practically-oriented economist. Schumpeter's complaint would have been seen by Peirce as barely coherent. Far from hiding one's hypotheses, as the charge of implicit theorizing suggests, Ricardo's radical simplification is the essence of analytical inference and essential to bringing mathematics *and* empirical evidence to bear on a scientific question in a systematic way. It is odd to simultaneously accuse Ricardo of having been dominated by immediate practical concerns and of having adopted a method that is criticized for being inadequate to those very practical concerns.

20 This comment comes from a letter, dated 18 April 1903, one of three to William James, written in the midst of the Harvard Lectures, on the subject of Achilles and the tortoise.

21 Peirce returned to insurance to illustrate points related to logic and probability throughout his scholarly life (e.g., W 2: 270, 1867; NEM II:48, 1896; NEM III/1: 239, 1909). His interest may have been prompted by his father's involvement in a dispute over the valuation of life insurance companies (B. Peirce 1859a, 1859b).

22 Peirce has changed notation here: earlier p had designated a probability; but now p designates a price and q a probability.

23 He explains his lack of elaboration, noting that the focus of the Harvard Lectures is not the theory of probability (EP 2: 137).

24 Peirce made a minor sign error in the equation for the optimal price, writing $p = ql + (\partial p/\partial n)n$, where a minus sign between the two terms on the right-hand side is required, as in equation (3). Peirce's sign error propagates through his algebra and is corrected *mutatis mutandis* in equation (4) and in the

quotation that immediately follows it. The corrections do not alter the the logic of his argument.

25 Two weeks before the Harvard Lectures, William James, who had set them up, noted that "[w]hatever you may say ... will be sure to catch and excite the best sort of man, and to pass over the heads of the others.... Naturally, I am myself most interested in your more concrete evolutionary ideas, for I have a bad head for logic and mathematics" (James 1903–1905 [2002]: 211). Two weeks after the lectures concluded, James suggested to Peirce that he revise them to be delivered at the Lowell Institute later in the year and advised him to "keep the ignoramus in view as your auditor" (James 1903–1905 [2002]: 258). A month later, commenting on Peirce's draft of an article aimed at a popular audience, James commented that "your mind inhabits a technical logical thicket of its own into which no other mind has yet penetrated, and *into* which you can't introduce them in popular articles ..." (James 1903–1905 [2002]: 281–282). In 1911, Peirce would reminisce about his extensive discussions on conceptions of infinity with James and how difficult it was for him to accept that Achilles would catch the tortoise (CP 6.182–184).

26 Recall that $\partial p / \partial n < 0$.

27 A search on JSTOR turns up four papers that discuss Peirce's primipostnumeral multitude but none that discuss the syllogism. A search on Google Books turns up only one actual reference to the syllogism—namely, volume 4 of the *Collected Papers*. It does, however, turn up several books that refer to *primipostnumeral*, but not to the *primipostnumeral syllogism*, as well as a larger number that do not refer to either term. The common thread among this latter group seems to be that they appear in our references (e.g., Kelly Parker, *The Continuity of Peirce's Thought* (Vanderbilt University Press, 1998)) or in the references to another of our papers on Peirce (e.g., Douglas Irwin, *Clashing over Commerce: A History of Trade Policy* (University of Chicago Press, 2017)). Although we appear to have discovered an odd feature of Google Books' search algorithm, we have confirmed that previous scholars have taken no notice of the primipostnumeral syllogism.

28 k here includes wages paid to workers, purchases of necessary physical inputs, such as fertilizer, and the interest on the monetary capital tied up in the production of the crop. Thus, φ is what modern economists refer to as "supernormal profits"—i.e., profits over and above the usual rate of return on money lent at interest. An irony, of which Ricardo is fully aware, is that, although supernormal profits are the farmer's (capitalist's) motive for expanding production, in equilibrium they are zero, having been fully converted into the landlord's rent.